specific range. Currently, the concept is being flight tested at the Naval Air Test Center to determine flying qualities and fuel savings potential. Design and development efforts leading to an engineering change proposal will be undertaken as the next step.

Several aerodynamic fairings aimed at reducing the total drag of the A-7 aircraft have been identified. Three small fairings which are currently under consideration for implementation reduce drag on the tailhook, gun port, and air-conditioning overboard dump. The pylon cavity fairing for unused pylons remains in the RDT&E phase at the present time. As part of this RDT&E, a single fairing design, common to all wing pylons, will be tested to examine feasibility and practicality of such an approach. Concurrent with the static installation evaluation, a wind-tunnel test will be conducted to establish the potential drag benefits.

The TF41 low-pressure turbine blade and vane replacement modification will reduce thrust specific fuel consumption by approximately 2-2½%. This estimate is based on analytical studies which parametrically varied turbine efficiency. The RDT&E effort consists of a full-scale performance proof-of-concept rig test which will be followed by a prototype flight test. In addition to fuel saving, this concept also offers reliability and maintainability improvements due to the resulting lower turbine inlet temperature requirements.

Removal of unused equipment is a recommended operational procedure throughout the Navy. Such removals yield immediate fuel saving through weight reduction (for internal equipment), and through weight and drag reduction (for external equipment). The A-7 Fuel Conservation Program specifically addresses the feasibility and practicality of frequent removal of wing pylons. Wing pylon removal offers significant drag reduction (10-20 counts) at no cost (excluding personnel cost) to the Navy. The recently completed RDT&E phase included engineering design reviews, analytical studies to compute fuel savings, and extensive discussions with A-7 maintenance and flight personnel.[3]

The onboard flight performance advisory system (FPAS) is a simplified extension of the computerized thrust/fuel management system used extensively by commercial airlines. The FPAS being considered for the A-7E aircraft[4] will utilize available memory space within the TC-2A tactical computer, and the existing cockpit display. This is a valid, cost-effective approach since the military qualified TC-2A is already installed in the aircraft and integrated with the air data computer. The only modification required is the revision of the TC-2A software tape to include the FPAS algorithm. In accordance with current plans, an A-7E FPAS algorithm will be developed and incorporated into the TC-2A tape during CY 1982. This effort will be followed by a fleet test and evaluation of the FPAS program's utility, accuracy, and contribution toward fuel conservation.

The stand-alone flight performance advisory systems, functionally similar to the onboard FPAS, are not integrated into the aircraft. Two stand-alone systems are currently being considered for the A-7E. The first is a desk-top minicomputer, similar to the HP-45C or the INTEL 8086, to be used for complete mission planning. The second is a pocket-sized programmable calculator, similar to the HP-41C,[5] to be used inflight for mission segment data. All FPAS configurations will use algorithms developed from a single performance data source. Fleet evaluation of the pocket-sized FPAS has demonstrated practicality as well as fuel savings.

Concluding Remarks

The A-7 study results led to two primary conclusions: first, the fuel savings from drag reduction and engine specific fuel consumption reduction modifications are large, and the payback potential is significant; and second, the fuel savings from weight reduction modifications are small, and the payback potential is nil.

The NAFC program has found similar results for other aircraft. The most cost effective (fuel savings exceed implementation cost) concepts are of the type illustrated by the A-7, i.e., drag improvements rather than airframe modifications, and more efficient operations through unused bomb rack/pylon removal and FPAS.

The active, ongoing fuel conservation program of the U.S. Navy has been briefly outlined. The systematic process of determining cost-effective, current-inventory aircraft modifications and operational procedure changes leading to improved flight-hours per gallon of fuel consumed has been described. The current status of the items under development for the A-7 aircraft is given. Positive demonstrations will result in the recommendation for and incorporation of modifications which will meet the schedule and fuel conservation goals for Navy A-7 fleet aircraft. The additions of the operational changes will provide fuel conservation in excess of energy program goals.

References

[1] "Department of the Navy Energy Plan FY 1981," OPNAV 41P4.B, Sept. 1980.

[2] Mallett, W.E., "A-7 Fuel-Saving Technology Applications Study," NADC-79015-60, Feb. 1980.

[3] Mallett, W.E., "A-7 Pylon Removal/Modification Study for Fuel Conservation," NADC 79293-60, Aug. 1980.

[4] Mallett, W.E., "A-7E Computerized Flight Performance Advisory System—Feasibility and Implications Investigation," NADC-80068-80, March 1981.

[5] Herskovitz, M., "The A-7E/HP41-C Pocket-Sized Flight Performance Advisory System," NADC-80241-60, Dec. 1980.

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Expansion Series of Integral Functions Occurring in Unsteady Aerodynamics

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Introduction

IN the kernel of singular integral equations for subsonic unsteady lifting surfaces (see, e.g., Ref. 1), the following real integral functions occur with arguments k, r, and X, which correspond to the reduced frequency, spanwise distance, and modified coordinate in the flow direction, respectively.

$$B_R^{(\nu)}(k,r,X) = \int_{-\infty}^{X} \frac{\cos kv}{(v^2+r^2)^{\nu+1/2}} \mathrm{d}v \tag{1}$$

$$B_I^{(\nu)}(k,r,X) = \int_{-\infty}^{X} \frac{\sin kv}{(v^2+r^2)^{\nu+1/2}} \mathrm{d}v \tag{2}$$

Two of the arguments, k and r, are assumed non-negative. For coplanar surfaces, the parameter ν takes the value of 1. For nonplanar geometry of lifting surfaces,[2] the case $\nu=2$ occurs as well as $\nu=1$. If the flow is supersonic, the lower limits of these integrals are interrupted by the Mach cone and therefore have finite values, as can be seen in Ref. 3. Integral functions Eqs. (1) and (2) are evaluated in most existing lifting

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surface theories (e.g., Ref. 4) in the form

$$I=\frac{1}{r^{2\nu}}\int_{-\infty}^{X/r}\frac{e^{ikru}}{(u^2+1)^{\nu+1/2}}\mathrm{d}u \tag{3}$$

Furthermore, the interval of integration may be divided into two subintervals $(-\infty,0)$ and $(0, X/r)$. Davies[5] has developed a code for computation to calculate the integrals of fixed intervals $(-\infty,0)$ by expanding them in Chebyshev polynomials. Desmarais[6] has also proposed a method to evaluate the imaginary part of Eq. (3) with $X=0$.

For variable limits X/r, Schwarz[7] has given an expression for the integral function Eq. (3) with $\nu=1$ in terms of the cylinder functions. Compared with Eqs. (1) and (2), Eq. (3) appears to have an advantage as the number of parameters has been reduced to two, i.e., X/r and kr. This integral form, however, possesses a numerical instability for small r, especially for negative X. As can easily be seen, it never works for a finite and negative X when $r=0$, although the integrals are finite. This difficulty was avoided for only the case $\nu=1$ in Ref. 8 by expanding Eqs. (1) and (2) directly. The resulting series has been applied by Ueda and Dowell[9] to the calculation of the unsteady aerodynamic loadings using the doublet point method.

The purpose of the present Note is to present a series for general values of non-negative integer ν in order to compute efficiently the integral functions Eqs. (1) and (2), avoiding any approximation or numerical quadrature.

Expansion Form

Each integral of Eqs. (1) and (2) is first divided into two parts,

$$B_R^{(\nu)}=\int_0^X\frac{\cos kv}{(v^2+r^2)^{\nu+1/2}}\mathrm{d}v+\int_0^\infty\frac{\cos kv}{(v^2+r^2)^{\nu+1/2}}\mathrm{d}v \tag{4}$$

$$B_I^{(\nu)}=\int_0^X\frac{\sin kv}{(v^2+r^2)^{\nu+1/2}}\mathrm{d}v-\int_0^\infty\frac{\sin kv}{(v^2+r^2)^{\nu+1/2}}\mathrm{d}v \tag{5}$$

These two parts of each equation are expanded individually in infinite series. The second parts of Eqs. (4) and (5) can be given by the modified Bessel functions and the modified Struve function,[10] respectively. The infinite series of the first parts are formed by term-by-term integration after the trigonometric functions in the integrands are replaced by their Maclaurin series. The series expansion thus obtained by $B_R^{(\nu)}$ contains infinitely increasing terms as r decreases which cancel out each other if X is negative. These terms must be eliminated analytically before the series is applied to numerical computations since they otherwise would cause a numerical instability.

After some manipulation to do this, the infinite series are obtained as follows:

$$B_R^{(\nu)}=\sum_{n=0}^{\infty}(-1)^nU_{2n}^{(\nu)}+\frac{(-1)^\nu}{(2\nu-1)!!}\frac{k^{2\nu}}{2^\nu}\sum_{n=0}^{\infty}\frac{(kr/2)^{2n}}{n!(n+\nu)!}$$
$$\times\left(\sum_{m=1}^{n}\frac{1}{m}+\sum_{m=n+1}^{n+\nu}\frac{1}{2m}-\gamma-\ln\frac{k}{2}\right) \tag{6}$$

$$B_I^{(\nu)}=\sum_{n=0}^{\infty}(-1)^nU_{2n+1}^{(\nu)}-\frac{(-1)^\nu\pi}{2(2\nu-1)!!}\left(\frac{k^{2\nu}}{2^\nu}\right)$$
$$\times\sum_{n=0}^{\infty}\frac{(kr/2)^{2n}}{n!(n+\nu)!} \tag{7}$$

where $U_m^{(\nu)}(X)$ $(m=1,2,...)$ is given by the recursion formula.

$$U_m^{(\nu)}=\frac{k(kX)^{m-1}}{(m-2\nu)m!(X^2+r^2)^{\nu-1/2}}-\frac{(kr)^2}{(m-2\nu)m}U_{m-2}^{(\nu)}\qquad m\neq 2\nu \tag{8}$$

The symbol of double factorials is used to define

$$(2n-1)!!=(2n-1)(2n-3)(2n-5)...3\cdot 1\qquad (-1)!!=1 \tag{9}$$

The summations with respect to m in Eq. (6) are taken to be 0 if the final integer of the summation is less than 1, and γ denotes Euler's constant. The initial terms for the recursion formula, Eq. (8), are written as

$$U_1^{(\nu)}=-\frac{k}{(2\nu-1)(X^2+r^2)^{\nu-1/2}} \tag{10}$$

$$U_{2n}^{(\nu)}=\frac{(\nu-n-1)!k^{2n}}{(2n)!r^{2(\nu-n)}}$$
$$\times\left[\sum_{m=0}^{\nu-n-1}\frac{(-1)^m}{(2m+2n+1)m!(\nu-m-n-1)!}\right.$$
$$\left.\times\left(\frac{X}{\sqrt{X^2+r^2}}\right)^{2m+2n+1}+\frac{(2n-1)!!}{2^n}\frac{2^{\nu-1}}{(2\nu-1)!!}\right]\qquad n<\nu \tag{11}$$

$$U_{2\nu}^{(\nu)}=-\frac{k^{2\nu}}{(2\nu)!}\left\{\sum_{m=1}^{\nu}\frac{X^{2m-1}}{(2m-1)(X^2+r^2)^{m-1/2}}\right.$$
$$\left.+\ln(\sqrt{X^2+r^2}-X)\right\} \tag{12}$$

Although Eq. (11) appears to have a singularity with respect to r, it is finite as r tends to zero if $X<0$. This can be proved if one uses the relationship

$$\sum_{l=0}^{n}\frac{(-1)^l n!}{(2l+2m+1)(n-l)!l!}=\frac{2^n n!(2m-1)!!}{(2n+2m+1)!!} \tag{13}$$

As an example, the $U_{2n}^{(\nu)}$ $(n<\nu)$ for $\nu=2$ are shown below.

$$U_0^{(2)}=\frac{2\sqrt{X^2+r^2}-X}{3(X^2+r^2)^{3/2}(\sqrt{X^2+r^2}-X)^2} \tag{14}$$

$$U_2^{(2)}=\frac{k^2}{6}\left[\frac{X^2}{(X^2+r^2)^{3/2}(\sqrt{X^2+r^2}-X)}+\frac{1}{X^2+r^2}\right] \tag{15}$$

These series, Eqs. (6) and (7), coincide exactly with the following known functions[10] when $X=0$:

$$B_R^{(\nu)}(k,r,0)=\frac{1}{(2\nu-1)!!}\left(\frac{k}{r}\right)^\nu K_\nu(kr) \tag{16}$$

$$B_I^{(\nu)}(k,r,0)=\frac{\pi}{2(2\nu-1)!!}\left(-\frac{k}{r}\right)^\nu\{I_\nu(kr)-L_{-\nu}(kr)\} \tag{17}$$

Alternative Form for Large Arguments

It is anticipated that in actual computation the series Eqs. (6) and (7) would require a rather large number of terms when the arguments are large numbers in absolute value. For this case of large arguments, an alternative series can be used. If we define the following representation of two functions as

$$\begin{Bmatrix}F_c^\nu(v)\\F_s^\nu(v)\end{Bmatrix}=\sum_{m=0}^{N}(-1)^m k^{-2(m+1)}\left(kf_\nu^{(2m)}(v)\begin{Bmatrix}\sin kv\\-\cos kv\end{Bmatrix}\right.$$

$$+f_{\nu}^{(2m+1)}(v) \begin{Bmatrix} \cos kv \\ \sin kv \end{Bmatrix} \Bigg) \tag{18}$$

where

$$f_{\nu}^{(n)}(v) = \frac{d^n}{dv^n}(v^2+r^2)^{-(2\nu+1)/2} \tag{19}$$

then asymptotic expansions of the functions $B_R^{(\nu)}$ and $B_I^{(\nu)}$ can be obtained by integrating by parts as

$$B_R^{(\nu)} \sim F_c^{\nu}(X) + c \tag{20}$$

$$B_I^{(\nu)} \sim F_s^{\nu}(X) \tag{21}$$

where c is an integration constant,

$$\begin{aligned} c &= \frac{2}{(2\nu-1)!!}\left(\frac{k}{r}\right)^{\nu} K_{\nu}(kr) \qquad X>0 \\ &= 0 \qquad\qquad\qquad\qquad\qquad X<0 \end{aligned} \tag{22}$$

The derivative, Eq. (19), can be calculated by the recursion formula

$$\begin{aligned} &(v^2+r^2)f_{\nu}^{(n+2)} + (2\nu+2n+3)vf_{\nu}^{(n+1)} \\ &\quad + (n+1)(2\nu+n+1)f_{\nu}^{(n)} = 0 \end{aligned} \tag{23}$$

Concluding Remarks

Appropriate series for numerical computation of the integral function Eqs. (1) and (2) have been obtained as Eqs. (6) and (7). These series hold except in the true singular domain $r=0$, $X \geq 0$ for $B_R^{(\nu)}$, and $r=X=0$ for $B_I^{(\nu)}$. Even in the proximity of singularities, however, the series are tractable because every singular part occurs explicitly in the initial terms, which is a favorable characteristic if one considers their application to the aerodynamic theory. It is worthy of notice that only the first of the two summations in $U_m^{(\nu)}$ is needed for Eqs. (6) and (7) if the lower limit of the integral Eqs. (1) and (2) is finite as is the case for supersonic flow.

The asymptotic expansions (20) and (21) can be used for large arguments. Numerical examples for $\nu=1$ are found in Ref. 8.

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References

[1] Dowell, E.H., Curtiss, H.C.Jr., Scanlan, R.H., and Sisto, F., *A Modern Course in Aeroelasticity,* Sijthoff and Noordhoff, the Netherlands, 1978.

[2] Landahl, M.T., "Kernel Function for Nonplanar Oscillating Surfaces in a Subsonic Flow," *AIAA Journal,* Vol. 5, May 1967, pp. 1045-1046.

[3] Harder, R.L. and Rodden, W.P., "Kernel Function for Nonplanar Oscillating Surfaces in Supersonic Flow," *Journal of Aircraft,* Vol. 8, Aug. 1971, pp. 677-679.

[4] Watkins, C.E., Woolston, D.S., and Cunningham, H.J., "A Systematic Kernel Function Procedure for Determining Aerodynamic Forces on Oscillating or Steady Finite Wings at Subsonic Speeds," NASA TR R-48, 1959.

[5] Davies, D.E., "Expansions in Series of Chebyshev Polynomials of a Function Occurring in Linear Unsteady Aerodynamic Theory," British ARC, R&M 3840, 1977.

[6] Desmarais, R.N., "Accurate Numerical Evaluation of Modified Struve Functions Occurring in Unsteady Aerodynamics," *Journal of Aircraft,* Vol. 16, July 1979, pp. 441-447.

[7] Schwarz, L., "Investigation of Some Functions Related to the Cylinder Functions of Zero Order," *Luftfahrtforschung,* Vol. 20, No. 12, 1944, pp. 341-372.

[8] Ueda, T., "Asymptotic Expansion of the Kernel Function in Subsonic Unsteady Lifting Surface Theory," *Journal of the Japan Society for Aeronautical Space Sciences,* Vol. 20, No. 326, 1981, pp. 169-174.

[9] Ueda, T. and Dowell, E.H., "A New Solution Method for Lifting Surfaces in Subsonic Flow," *AIAA Journal,* Vol. 20, March 1982, pp. 348-355.

[10] Abramowitz, M. and Stegun, I.A., *Handbook of Mathematical Functions,* Dover, New York, 1965.

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Evolution of Swirl in Two-Dimensional-Nozzle Flow

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In a typical turbine engine, the residual swirl in the exhaust gases, though somewhat detrimental to performance, has been recognized as a mechanism that favors mixing of the exhaust.[1,2] In an axisymmetric nozzle plume, increased mixing due to shear tends to lower the plume temperature somewhat and hence contributes to some reduction of the jet noise and IR signature. As interest in two-dimensional (2D) nozzles continues to rise, some questions about the behavior of engine swirl in 2D nozzles are raised: What happens to the engine swirl going through a rectangular flow passage? And how does this affect mixing in the plume? This Note reports some simple studies in Northrop's water tunnel[3] which provide preliminary answers to these questions.

In the water-tunnel experiments, a plastic duct (Fig. 1) was aligned with the flow in the center of the water tunnel. Water entered through a circular bellmouth and duct and then passed to a rectangular nozzle via a transition section. The exit area was made equal to the circular area to avoid separation. One or more plastic pinwheels with blade angles up to 22.5 deg were placed upstream in the circular section to produce a swirling flow. Flow paths were observed through dyes released from tubes within the duct and from probes in the external flow outside of the nozzle. Two models with rectangular exits of aspects ratios (Æ) 4 and 10 were tested. The tunnel velocity was approximately 0.7 ft/s. The pressure losses produced by the pinwheel in this arrangement gave somewhat lower nozzle exit velocities than those in the external flow; however, the phenomena observed are expected to be typical for jets of higher velocities. For clarity, results are presented for a blade angle of 22.5 deg, which was the maximum tested.

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